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Additive Slacks-Based Measure of Efficiency for Dealing with Undesirable Outputs Based on DEA-R Model

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Abstract

In this paper, we present an Additive Slacks-Based Measure (ASBM) model for measuring efficiency Decision-Making Unit (DMU) in the presence of undesirable outputs based on DEA Ratio based (DEA-R) model. In order to obtain a reasonable measure of efficiency, this paper proposes a concept for determining the minimum number of undesirable outputs that a DMU is allowed to generate based on the assertion of weak disposability. We show the effect of producing excessive amounts of undesirable outputs on efficiency. We propose an alternative form of the ASBM model to measure efficiency based on DEA-R models. In the first stage, we introduce counterpart (hypothetic) units corresponding to each DMUs. We obtain the true efficiency scores and slack variables regarding the input and output components of each of the DMUs. In the second stage, we obtain the efficiency scores in the presence of undesirable outputs based on ASBM-DEA-R model. In the following, we illustrate the proposed approach with a numerical example. At the end, the results of the research are given.

Keywords: Data envelopment analysis, DEA-R, Ratio data, Undesirable output, Efficiency, Non-radial DEA, ASBM.

1 | Introduction

Recently, various management systems have been proposed to evaluate the environmental impact of products or manufacturing processes. Due to the unavoidable circumstances of emission of pollutants during the production process of industrial activities, environmental management assumes particular importance in the manufacturing sector. But, under the current production conditions, the pollutants cannot be ignored. For instance, the production of fabric or yarn in a textile company is always associated with water pollution as a result of the dyeing process. In this paper, we use ASBM DEA-R model to measuring efficiency that takes into consideration the undesirable aspects of operational assessment of the Decision-Making Unit (DMU) environmental performance. We consider 'Desirable output' as the preferred product of the DMU and 'undesirable output' as unfavorable aspect of the production output.

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Scheel [1] classified the methods of measuring the efficiency of DMUs in the presence of undesirable outputs into two categories, direct and indirect. Direct methods are closely related to the concept of Weak disposability [2] and [3]. Indirect methods are including inverse input methods, additive inverse, translated inverse, and multiplicative inverse methods. Zhou et al. [4] performed a classification by examining DEA techniques in energy and environmental studies, and categorized the direct methods that work on the original data as, the Slacks-Based Measure (SBM), the Directional Distance Function (DDF), and hyperbolic models in Kao [5].

Also, Song et al. [6] divided the methods of environmental efficiency evaluation in the presence of undesirable outputs into input reverse, data transformation, and disposability-related methods. Dakpo et al. [7] provide an extensive review of papers on performance benchmarking in the presence of undesirable outputs, they divided methods into five approaches: free disposability of the inputs, data transformation, weak disposability of the undesirable outputs, materials balance principles (including the weak G-disposability), and two sub-technologies (including the by-production model of Murty et al. [8] and natural and managerial disposability as proposed in Sueyoshi and Goto [9]). Kao and Hwang [10] proposed a model for measuring the effect of undesirable outputs on efficiency measurement of DMUs.

The several DEA models are proposed upon input and output slacks. Charnes et al. [11] proposed the additive DEA model that the DEA objective function is defined in the form of the summation of all input and output slacks. The additive model can identify the efficient DMUs, but it cannot be used to produce a comparable DEA score. To face this limitation of the additive model, Green et al. [12] developed the additive DEA model into an Additive Slacks-Based Measure (ASBM) that this model obtained a DEA efficiency score between. The proposed ASBM model by Green et al. [12] is a nonlinear programming problem. Chen and Zhu [13] developed ASBM model and presented the ASBM in a form that gets efficiency score between zero and unity directly. They show that the additive slacks-based model can be applied to modelling network DEA where the internal structures of DMUs is considered. The additive slacks-based network DEA can be solved using Second Order Cone Programming (SOCP). They show that the additive slacks-based approach can obtain divisional efficiencies in the form of Pareto optimal. Gerami et al. [14] proposed the ASBM based on input and output slacks and show that ASBM model can linearize using multi-objective programming. They develop value efficiency analysis based on the ASBM model.

DEA-R models first introduced in Despić et al. [15] that combines DEA and ratio analysis, and then, such models have been applied by many other researchers. The convexity condition is one of the axioms in basic DEA models. Emrouzinejad and Amin [16] proposed a new convexity assumption as well as enhancements to basic DEA models to encounter with this problem. Wei et al. [17]-[19] extended the DEA-R models in new directions. They developed relations between traditional DEA models and ratio-based DEA-R models. Mozaffari et al. [20] and [21] proposed DEA and DEA-R models based on cost and revenue efficiency concept. Olesen et al. [22] and [23], Hatami-Marbini and Toloo [24] shown the problems with ratio data after classifying them, defined a production possibility set and introduced the corresponding models in constant/variable returns to scale technology. Mozaffari et al. [25] introduce a DEA-R production possibility set under the assumption of constant returns to scale technology and propose a method for identifying DEA-R-efficient surfaces. Mozaffari et al. [25] have examined efficient hyperplanes in DEA-R. In addition, Gerami et al. [26] proposed multi-criteria ratios for two-stage network. Finally, Gerami et al. [27] developed a novel network DEA-R model for evaluating hospital services supply chain performance. In this paper, we present an ASBM model for measuring efficiency DMU in the presence of undesirable outputs based on DEA ratio based (DEA-R) model.

The rest of this paper is organized as follows. In the Section 2, we present an alternative form of the ASBM model to measure efficiency based on DEA-R model. In the tread section, we a two-stage process based on ASBM DEA-R model for measuring efficiency DMU in the presence of undesirable outputs. The Section 4 describes the proposed approach with a simple numerical example, at the end we bring the results of the research.

2 | Preliminary

In this section, we present the DEA-R and ASBM DEA-R models in variable returns to scale technology.

2.1 | DEA-R Model

Suppose we have n decision units as $DMU_j=\left(x_j,y_j\right), j=1,...,n$. The input and output vectors corresponding to $DMU_j=\left(x_j,y_j\right), j=1,...,n$ as $x_j=\left(x_{1j},...,x_{mj}\right)$ and $y_j=\left(y_{1j},...,y_{sj}\right)$. We suppose that $x_{ij}>0,\ \ y_{rj}>0,\ \ i=1,...,m,\ \ r=1,...,s,\ \ j=1,...,n.$ Suppose the ratios, $\frac{x_{ij}}{y_{rj}}>0, i=1,...,m, r=1,...,s.$, $j=1,...,n,$ in the input orientation are defined. Supposed $s_{ir}, i=1,...,m, r=1,...,s,$ show the variabl slacks relate to the ratio of the i-th input component to the r-th output component corresponds to $DMU_j=\left(x_j,y_j\right), j=1,...,n$.

The DEA-R model in the input orientation in the envelopment form is as follows [10].

$$
\begin{aligned}
\theta^*_{DEA-R} = \min\ & \theta_{DEA-R} - \varepsilon\left(\sum\nolimits_{i=1}^{m}\sum\nolimits_{r=1}^{s} s_{ir}\right),\\
\text{s.t.}\quad & \sum\nolimits_{j=1}^{n}\lambda_j\left(\frac{x_{ij}}{y_{rj}}\right)+s_{ir}=\theta_{DEA-R}\left(\frac{x_{io}}{y_{ro}}\right), i=1,...,m, r=1,...,s,\\
& \sum\nolimits_{j=1}^{n}\lambda_j=1,\quad \lambda_j\geq 0, j=1,...,n, \varepsilon \text{ is non}-\text{Archimedean},\\
& s_{ir}\geq 0, i=1,...,m, r=1,...,s.
\end{aligned}
\tag{1}
$$

Definition 1. DMU_o is called (weakly) DEA-R efficient in evaluation with *model (1)* if and only if $\theta^*_{DEA-R}=1.$

Definition 2. DMU_o is called strong DEA-R efficient in evaluation with *model (1)* if and only if $\theta^*_{DEA-R}=1,\ \ s_{ir}=0, i=1,...,m,\ \ r=1,...,s.$

2.2 | ASBM DEA-R Model

We suppose that $x_{ij}>0,\ \ y_{rj}>0,\ \ i=1,...,m,\ \ r=1,...,s, j=1,...,n.$ Suppose the ratios, $\frac{x_{ij}}{y_{rj}}>0, i=1,...,m, r=1,...,s.$, $j=1,...,n,$ is the ratio of the i-th input component to the r-th output component corresponds to $DMU_j=\left(x_j,y_j\right), j=1,...,n$. We present the ASBM-DEA-R model based on the ratio of input components to output components as follows:

$$\theta^*_{ASBM\,DEA-R} = \min \frac{1}{m \times s}\left(\sum_{i=1}^{m}\sum_{r=1}^{s}\frac{\left(\frac{x_{io}}{y_{ro}}\right)-s_{ir}}{\left(\frac{x_{io}}{y_{ro}}\right)}\right),$$
$$\begin{aligned} \text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j\left(\frac{x_{ij}}{y_{rj}}\right)+s_{ir}=\left(\frac{x_{io}}{y_{ro}}\right), i=1,\dots,m, r=1,\dots,s, \\ & \sum_{j=1}^{n}\lambda_j=1, \quad \lambda_j \geq 0, j=1,\dots,n, \\ & s_{ir}\geq 0, i=1,\dots,m, r=1,\dots,s. \end{aligned} \tag{2}$$

Supposed $\left(\lambda^*, s^*\right)$ is an optimal solution of *model (2)*.

Definition 2. DMU_o is called ASBM DEA-R-efficient in evaluation with *model (1)* if and only if $\theta^*_{ASBM\,DEA-R}$ or equivalent $s^*_{ir}=0, \; i=1,\dots,m, r=1,\dots,s,$ otherwise DMU_o will be ASBM-DEA-R-inefficient. We define the efficiency score based on *model (2)* as follows:

$$\frac{1}{m \times s}\left(\sum_{i=1}^{m}\sum_{r=1}^{s}\frac{\left(\frac{x_{io}}{y_{ro}}\right)-s^*_{ir}}{\left(\frac{x_{io}}{y_{ro}}\right)}\right).$$

3 | Measuring the Efficiency in the Presence of Undesirable Outputs Based on the ASBM DEA-R Model

In this section, we consider undesirable outputs as $u_j=\left(u_{1j},\dots,u_{dj}\right), d=1,\dots,D.$ Then, each of DMUs is as $DMU_j=\left(x_j, y_j, u_j\right), j=1,\dots,n.$ We first introduced the concept of a counterpart (hypothetic) unit corresponding to each of the DMUs, and presented the efficiency of a DMU based on the minimum level of undesirable output. We obtained this amount of efficiency corresponding to each of the DMUs based on the counterpart units. The efficiency scores of a production unit and its counterpart (hypothetic) unit with the minimum amount of undesirable output are measured based on ASBM DEA-R model.

The difference between these two efficiencies is then a measure of the effects of producing excessive amounts of the undesirable outputs on efficiency for a given production unit. Counterpart units have the same input and output levels as the original units, but their undesirable output level is lower than the original units, and to determine the level of undesirable outputs from these DMUs. assume that $p_d, d=1,\dots,D,$ are slacks corresponding to undesirable output components, we can solve the following model.

$$\begin{aligned} \max & \left(\sum_{d=1}^{D} p_d\right), \\ \text{s.t.} \quad & \sum_{j=1}^{n}\mu_j y_{rj}=y_{ro}, \; r=1,\dots,s, \\ & \sum_{j=1}^{n}\mu_j u_{dj}+p_d=u_{do}, d=1,\dots,D, \end{aligned} \tag{3}$$

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$$\sum_{j=1}^{n}\mu_j = 1, \quad \mu_j \ge 0, j = 1, \dots, n,$$
$$p_d \ge 0, \ d = 1, \dots, D.$$

Supposed (μ^*, p^*) that $p^* = (p_1^*, \dots, p_D^*)$ is an optimal solution of *model (3)*. In *model (3)*, the number of desirable outputs remains constant and the minimum level of undesirable outputs that DMU_o is allowed to generate is determined as follows:

$$s_{do}^{new} = u_{do} - p_d^*, d = 1, \dots, D.$$

We solve *model (3)*, for each of $DMU_j = (x_j, y_j), j = 1, \dots, n$, by considering them as under evaluation DMU. We determine counterpart (hypothetic) unit corresponding to for each of $DMU_j = (x_j, y_j), j = 1, \dots, n$, as follows:

$$DMU_j^{new} = (x_j, y_j, u_j^{new}), j = 1, \dots, n.$$

We suppose that $x_{ij} > 0, \ y_{rj} > 0, \ i = 1, \dots, m, \ r = 1, \dots, s, \ j = 1, \dots, n.$ Suppose the ratios, $\frac{x_{ij}}{y_{rj}} > 0, i = 1, \dots, m, r = 1, \dots, s., \ j = 1, \dots, n,$ is the ratio of the i-th input component to the r-th output component corresponds to $DMU_j = (x_j, y_j), j = 1, \dots, n$. Also, we suppose that $u_{dj}^{new} > 0, \ y_{rj} > 0,$ $d = 1, \dots, D, \ r = 1, \dots, s, \ j = 1, \dots, n.$ Suppose the ratios, $\frac{u_{dj}^{new}}{y_{rj}} > 0, \ d = 1, \dots, D, \ r = 1, \dots, s., \ j = 1, \dots, n,$ is the ratio of the d-th undesirableoutput component to the r-th output component corresponds to $DMU_j = (x_j, y_j), \ j = 1, \dots, n.$.

Suppose $s_{ir}, i = 1, \dots, m, r = 1, \dots, s,$ show the variabl slacks relate to the ratio of the i-th input component to the r-th output component corresponds to $DMU_j = (x_j, y_j), j = 1, \dots, n$. Also, assume that $t_{dr}, \ d = 1, \dots, D, \ r = 1, \dots, s,$ show the variabl slacks relate to the ratio of the d-th undesirable output component to the r-th desirable output component corresponds to $DMU_j = (x_j, y_j), j = 1, \dots, n$.

We present the ASBM-DEA-R model based on the ratio of input components to desirable output components and the ratio of undesirable output components to output components as follows:

$$\theta_{ASBM\,DEA-R}^{U} = \min \frac{1}{(m+D)\times s}\left(\left(\sum_{i=1}^{m}\sum_{r=1}^{s}\frac{\left(\frac{x_{io}}{y_{ro}}\right) - s_{ir}}{\left(\frac{x_{io}}{y_{ro}}\right)}\right) + \left(\sum_{d=1}^{D}\sum_{r=1}^{s}\frac{\left(\frac{u_{do}}{y_{ro}}\right) - t_{dr}}{\left(\frac{u_{do}}{y_{ro}}\right)}\right) \right), \quad (4)$$

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$$\text{s.t.} \quad \sum_{j=1}^{n}\lambda_j\left(\frac{x_{ij}}{y_{rj}}\right)+s_{ir}=\left(\frac{x_{io}}{y_{ro}}\right), i=1,\dots,m, r=1,\dots,s,$$
$$\sum_{j=1}^{n}\lambda_j\left(\frac{u_{dj}^{new}}{y_{rj}}\right)+t_{dr}=\left(\frac{u_{do}}{y_{ro}}\right), d=1,\dots,D, r=1,\dots,s,$$
$$\sum_{j=1}^{n}\lambda_j=1, \quad \lambda_j\geq 0, j=1,\dots,n,$$
$$s_{ir}\geq 0, \quad t_{dr}\geq 0, \quad i=1,\dots,m, \; d=1,\dots,D, \; r=1,\dots,s.$$

Supposed $\left(\lambda^*, s^*, t^*\right)$ is an optimal solution of *model (4)*, that $s^*=(s_{11}^*,\dots,s_{m+s}^*)$ and $t^*=(t_{11}^*,\dots,t_{D+s}^*)$.

Definition 3. DMU_o is called ASBM DEA-R-efficient in evaluation with *model (4)* if and only if $\theta_{ASBM\,DEA-R}^{U}=1$, or equivalent $s_{ir}^*=t_{dr}^*=0, \; i=1,\dots,m, \; d=1,\dots,D, \; r=1,\dots,s,$ otherwise DMU_o will be ASBM-DEA-R-inefficient with *model* (4).

We define the efficiency score based on *model (4)* as follows:

$$\frac{1}{(m+D)\times s}\left((\sum_{i=1}^{m}\sum_{r=1}^{s}\frac{\left(\frac{x_{io}}{y_{ro}}\right)-s_{ir}^*}{\left(\frac{x_{io}}{y_{ro}}\right)})+(\sum_{d=1}^{D}\sum_{r=1}^{s}\frac{\left(\frac{u_{do}^{new}}{y_{ro}}\right)-t_{dr}^*}{\left(\frac{u_{do}^{new}}{y_{ro}}\right)})\right).$$

4 | Numerical Example

This section illustrates the proposed approach using a numerical example. Consider 13 DMUs including an input and desirable output and an undesirable output according to *Table 1*. We consider the number of employees a single input and Gross Domestic Product (GDP) as desirable output and NO_x as undesirable output.

Table 1. Data set of original DMUs.

DMUs	Employees	GDP (DM)	NOx (1000t)
DMU01	3793	385.7	76
DMU02	35782	3457.4	612
DMU03	2601	248.2	61
DMU04	12027	802.4	151
DMU05	22057	2204	294
DMU06	25936	1579.3	456
DMU07	3821	163.8	33
DMU08	19943	1560.9	295
DMU09	1262	92.2	24
DMU10	6782	566.9	116
DMU11	4417	144	24
DMU12	4134	330.9	78
DMU13	2016	179.2	54

At first, we evaluate DMUs based on DEA-R model (*model (1)*), the efficiency scores are in the second column of *Table 3*. As can be seen, DMU1 is only DEA-R efficient DMU and the other DMUs are DEA-R in efficient. In the following, we evaluate DMUs based on ASBM DEA-R model (*model (2)*), the efficiency scores are in the thread column of *Table 3*. As can be seen, DMU1 is only ASBM DEA-R efficient DMU and the other DMUs are ASBM DEA-R in efficient. The results of *model*s *(1)* and *(2)* are similar. But this is not true in general. *Models (1)* and *(2)* obtain the efficiency score of each DMUs without considering the undesirable output. Now, in order to calculate the efficiency of each DMUs in the presence of undesirable outputs, we use of the proposed approach in this paper, in this way, at first, we obtain the undesirable outpouts of counterpart units based on *model (3)*. We solve *model (3)* and obtain the minimum level of undesirable output according to the desired input and output values. As stated in the thread section, we first solve *model (3)* and introduce the counterpart (hypothetic) units corresponding to each of the DMUs. Counterpart units have the same input and output levels as the original units, but their undesirable output level is lower than the original units, and to determine the level of undesirable outputs from these DMUs. The input, desirable output, and undesirable output values corresponding to counterpart units are listed in *Table 2* based on *model (3)*. Also, we evaluated the efficiency scores of counterpart units based on *model (2)* in the presence of undesirable outputs, the efficiency scores are given in the last column of *Table 2*.

Table 2. Data set of counterpart DMUs based on model (3) and the efficiency score based on model (2).

DMUs	Employees	GDP (DM)	NOx (1000t)	Efficiency
DMU01	3793	385.7	51.45	1
DMU02	35782	3457.4	461.196	0.9751
DMU03	2601	248.2	33.108	1
DMU04	12027	802.4	107.035	0.828
DMU05	22057	2204	294	0.9913
DMU06	25936	1579.3	210.669	0.7994
DMU07	3821	163.8	21.85	0.7108
DMU08	19943	1560.9	208.214	0.8898
DMU09	1262	92.2	12.299	0.8592
DMU10	6782	566.9	75.621	0.911
DMU11	4417	144	19.209	0.6603
DMU12	4134	330.9	44.14	0.8936
DMU13	2016	179.2	23.904	0.9569

Now, in order to evaluate the the efficiency of orginal DMU based on the counterpart units, we solve *model (4)*. *Model (4)* obtain the efficiency scores and slacks related to the input components and undesirable output components. This model achieves the true efficiency scores and slacks in the input components and undesirable output components of each original DMUs based on counterpart (hypothetic) units. The results are given in the tread column of *Table 3*. As can be seen, we have not ASBM DEA-R efficient DMUs based on the *model (4)*, and all units are ASBM DEA-R inefficient in the presence of undesirable outputs.

Table 3. The efficiency scores of models (1), (2), and (4).

DMUs	Eq (1)	Eq (2)	Eq (4)
DMU01	1	1	0.8385
DMU02	0.9502	0.9502	0.8519
DMU03	0.9384	0.9384	0.7406
DMU04	0.6561	0.6561	0.6825
DMU05	0.9826	0.9826	0.9913
DMU06	0.5988	0.5988	0.5304
DMU07	0.4216	0.4216	0.5418
DMU08	0.7697	0.7697	0.7378
DMU09	0.7185	0.7185	0.6155
DMU10	0.822	0.822	0.737
DMU11	0.3206	0.3206	0.5605
DMU12	0.7872	0.7872	0.6765
DMU13	0.8741	0.8741	0.6584

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In this paper, we evaluate the efficiency of DMUs based on ASBM and DEA-R models in the presence of undesirable outputs. For this purpose, we presented a new model under title ASBM DEA-R. model based on the ASBM and DEA-R models. In this way, we first obtain the optimal level of undesirable outputs based on their decrease based on the increase in desired outputs simultaneously. We present the new units as counter units. These units have the same levels of input and desirable output to their respective original units, but their undesirable output levels of counter units are lower than the level of undesirable outputs of their respective original units. Then, using the ASBM DEA-R model, we obtained the efficiency of DMUs. We have shown that if the ratio of input to output components is important to the decision maker, we can use the above models. The proposed models also use the inefficiency slack values corresponding to the input components and undesirable output components in the efficiency evaluation. These models obtain the true scores of efficiencies by considering the lowest level of undesirable outputs and the inefficiency slack values corresponding to the input components and undesirable output components. As future work we can develop the proposed approach based on the ASBM DEA-R model in mix form or in output oriented. Also, we can also extend the proposed approach to other data structures in DEA such as two-stage network structure or in the presence of fuzzy data.

Conflicts of Interest

Javad Gerami: Project administration, Software, Supervision, Writing - original draft, Writing - review & editing. Mohammad Reza Mozaffari: Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing - original draft, Writing - review & editing.

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