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Optimization of Fuzzy Linear Fractional Programming Problem with Fuzzy Numbers

Sapan Kumar Das

Department of Revenue, Ministry of Finance, New Delhi, India; cool.sapankumar@gmail.com.

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Abstract

In this paper, we consider a Fuzzy Linear Fractional Programming (FLFP) problem under the condition that the objective function and the values of the right-hand side are represented by symmetric trapezoidal fuzzy numbers while the left-hand side constraints are represented by real numbers. Decision variables are characterized by trapezoidal fuzzy numbers and non-negative numbers. Utilizing the ranking function and computation of trapezoidal fuzzy numbers, the FLFP problem is transformed into a Crisp Linear Fractional Programming (CLFP) problem. This paper outfits another idea for diminishing the computational complexity, in any case without losing its viability crisp LFP problem. A modified possibility programming problem, Swarup is utilized to solve this program. Lead from real-life problems, a couple of mathematical models is considered to survey the legitimacy, usefulness and applicability of our method. Finally, some mathematical analysis along with one case study is given to show the novel strategies are superior to the current techniques.

Keywords: Fuzzy Linear Fractional Programming (FLFP), Ranking, Simplex method, Symmetric trapezoidal number.

1 | Introduction

Linear Fractional Programming (LFP) problem is a mathematical technique for optimal allocation to several activities on the basis of given decision of optimality. LFP is one of the most important techniques which is applied in operation research. Till now, it has been extended more methods in order to apply in real world problems within framework of LFP problem. Linear fractional objectives have different applications in the field of finance, hospitality, military academy and foreign loans to actual loans. In real world problem sometimes, the decision is uncertainty at that time the researchers are introduced fuzzy optimization. Interactive fuzzy programming has been proposed the upper level and multiple decision fuzzy set theory has been used to handle imprecise data in LFP problem of membership in a set.

Corresponding Author: cool.sapankumar@gmail.com

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The theory of fuzzy programming was introduced by Tanaka et al. [1] based on fuzzy decision framework of Bellman and Zadeh [2] proposed the first formulation of linear programming problem in fuzzy environments. Lai and Hwang [3], Shaocheng [4], considered the situation where all parameters are fuzzy. Many researchers adopted this concept for solving Fuzzy Linear Fractional Programming (FLFP) problems [5], [6] and [7]. Dehghan et al. [8] proposed a fuzzy linear programming approach for finding the exact solution of fuzzy linear programming problems. Lotfi et al. [6] introduced a method to obtain the approximate the solution of fully fuzzy linear programming problems.

In this paper, a new method is proposed for finding the fuzzy optimal solution of FLFP problems with inequality constraints. The coefficients of the objective function and the values of the right-hand side are represented by symmetric trapezoidal fuzzy numbers while the left-hand side constraints are represented by real numbers. We introduce a new type of fuzzy arithmetic for symmetric trapezoidal fuzzy numbers and propose a method for solving FLFP problem with converting them to CLFP problems and then used by famous simplex type method by Swarup.

This paper is organized as follows: In Section 2 some basic definitions of fuzzy symmetric trapezoidal fuzzy number and some arithmetic results also. In Section 3, formulation of FLFP problems and application of ranking function for solving FLFP problems are established. A new method is proposed for solving FLFP problems in Section 4. In Section 5, we give a numerical example involving symmetrical trapezoidal fuzzy numbers to illustrate the theory developed in this paper.

2 | Preliminaries

The aim of this section is to present some notations and results of fuzzy set theory are discussed.

Definition 2.1. [5]. A convex fuzzy set $\tilde{A}$ on R is a fuzzy number if the following conditions hold:

a) Its membership function is piecewise continuous.
b) There exist three intervals [a, b], [b, c] and [c, d] such that $\mu_{\tilde{A}}$ is increasing on [a, b], equal to 1 on [b, c], decreasing on [c, d] and equal to 0 elsewhere.

Definition 2.2. [5]. The arithmetic operations on two symmetric trapezoidal fuzzy numbers

$\tilde{A} = (a^L, a^U, \alpha, \alpha)$ and $\tilde{B} = (b^L, b^U, \beta, \beta)$ are given by:

$$\tilde{A} + \tilde{B} = (a^L + b^L, a^U + b^U, \alpha + \beta, \alpha + \beta),$$

$$\tilde{A} - \tilde{B} = (a^L - b^L, a^U - b^U, \alpha - \beta, \alpha - \beta),$$

$$\tilde{A}\tilde{B} = ((\frac{a^L + a^U}{2})(\frac{b^L + b^u}{2}) - t, (\frac{a^L + a^U}{2})(\frac{b^L + b^U}{2}) + t, \left|a^U\beta + b^U\alpha\right|, \left|a^U\beta + b^U\alpha\right|),$$

Where

$$t = \frac{t_2 - t_1}{2}, \ t_1 = min\{a^L b^L, a^U b^U, a^U b^L, a^U b^U\}, t_2 = max\{a^L b^L, a^U b^U, a^U b^L, a^U b^U\}.$$

Das| Big. Data. Comp. Vis. 1(1), (2021) 30-35

$$k\tilde{A}=\begin{cases}(ka^L,ka^U,k\alpha,k\alpha) & k\geq 0\\ (ka^U,ka^L,-k\alpha,-k\alpha) & k<0.\end{cases}$$

Definition 2.3. [5]. Let $\tilde{A}=(a^L,a^U,\alpha,\alpha)$ and $\tilde{B}=(b^L,b^U,\beta,\beta)$ be two symmetric trapezoidal fuzzy numbers. The relations $\tilde{\leq}$ and $\approx$ are defined as follows:

a) $\frac{(a^L-\alpha)+(a^U-\alpha)}{2}<\frac{(b^L-\beta)+(b^U-\beta)}{2}$, that is $\frac{a^L+a^U}{2}<\frac{b^L+b^U}{2}$ (in this case, we may write $\tilde{A}\tilde{<}\tilde{B}$), or

b) $\frac{a^L+a^U}{2}=\frac{b^L+b^U}{2}, b^L<a^L, a^U<b^U$ (In this case we say $\tilde{A}\approx\tilde{B}$), or

c) $\frac{a^L+a^U}{2}=\frac{b^L+b^U}{2}, b^L=a^L, a^U=b^U, \alpha\leq\beta$ (In this case we say $\tilde{A}\approx\tilde{B}$).

Definition 2.4. [5] A fuzzy set $\tilde{A}$ on R is called a symmetric trapezoidal fuzzy number if its membership function is defined as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{x-(a^L-\alpha)}{\alpha} & a^L-\alpha\leq x\leq a^L,\\ 1 & a^L\leq x\leq a^U,\\ \frac{(a^u+\alpha)-x}{\alpha} & a^U\leq x\leq a^U+\alpha,\\ 0 & \text{else.}\end{cases}$$

3 | General Form of FLFP Problem

In the proposed method, it is assumed that the general form of FLFP problems as follows:

$$\begin{aligned}&\text{Max } Z=\frac{\tilde{c}^t x+\tilde{\alpha}}{\tilde{d}^t x+\tilde{\beta}}\\ &\text{s.t.}\quad Ax\leq\tilde{b},\\ &\qquad x\geq 0.\end{aligned}\qquad (p1)$$

Where $\tilde{c}^t$, $\tilde{d}^t$, $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{b}$ symmetric trapezoidal fuzzy numbers and others are real numbers.

However, if all the parameters of (p_1) are obtained by replacing CLFP problem.

4 | Proposed FLFP Method

According to Definition 2.3, we define a rank for each symmetric trapezoidal fuzzy number for comparison purposes. Assuming that $\tilde{A}=(a^L,a^U,\alpha,\alpha)$ is a symmetric trapezoidal fuzzy number, then $R(\tilde{A})=\frac{a^L+a^U}{2}$. This equation allows us to convert the FLFP problem in to a Crisp Linear Fractional Programming (CLFP) problem. We substitute the rank order of each fuzzy number for the corresponding fuzzy number in the fuzzy problem under consideration. This leads to an equivalent CLFP problem which can be solved by standard method.

Next, we solve an example by our method.

Example we find the fuzzy optimal solution of the following problem.

$$\begin{aligned}
&\text{Max } \tilde{Z}=\frac{(4,6,2,2)x_1+(2,4,3,3)x_2}{(4,6,3,3)x_1+(1,3,2,2)x_2}\\
&\text{s.t. } \quad 3x_1+5x_2\leq(13,15,6,6),\\
&\qquad 5x_1+2x_2\leq(9,11,5,5),\\
&\qquad x_1,x_2\geq 0.
\end{aligned}\tag{1}$$

We first substitute the rank order of each number for its corresponding fuzzy number in the above FLFP problem to obtain the following crisp linear fractional problem:

$$\begin{aligned}
&\text{Max } \tilde{Z}=\frac{5x_1+3x_2}{5x_1+2x_2+1}\\
&\text{s.t. } \quad 3x_1+5x_2\leq 15,\\
&\qquad 5x_1+2x_2\leq 10,\\
&\qquad x_1,x_2\geq 0.
\end{aligned}\tag{2}$$

We then construct the standard form of the *Eq. (2)* as follows where x_3,x_4 the slack variables are

$$\begin{aligned}
&\text{Max } \tilde{Z}=\frac{5x_1+3x_2}{5x_1+2x_2+1}\\
&\text{s.t. } \quad 3x_1+5x_2+x_3=15,\\
&\qquad 5x_1+2x_2+x_4=10,\\
&\qquad x_1,x_2,x_3,x_4\geq 0.
\end{aligned}\tag{3}$$

After computing $z_j^1-c_j, z_j^2-d_j, \Delta_j$ and the usual matrix Y, the initial basics feasible solution is given in the following table:

Table 1. Initial Iteration. Introduce y_1 and drop y_4.

			y_1	y_2	y_3	y_4	
		c_j	5	3	0	0	
		d_j	5	2	0	0	
d_b	c_b	x_b	y_1	y_2	y_3	y_4	min ratio
0	0	$y_3 = 15$	3	5	1	0	5
0	0	$y_4 = 10$	5	2	0	1	2
$z^2 = 1$	$z^1 = 0$	z=0					
		$z_j^1 - c_j$	-5	-3	0	0	
		$z_j^2 - d_j$	-5	-2	0	0	
		Δ_j	-5	-3	_	_	

Table 2. First Iteration. Introduce y_2 and leave y_3.

d_b	c_b	x_b	y_1	y_2	y_3	y_4	min ratio
0	0	$y_3 = 9$	0	19/5	1	-3/5	45/19
5	5	$x_1 = 2$	1	2/5	0	1/5	5
$z^2 = 11$	$z^1 = 10$	z=10/11					
		$z_j^1 - c_j$	0	-1	0	1	
		$z_j^2 - d_j$	0	0	0	1	
		Δ_j	_	-11	_	1	

Table 3. Second Iteration. Introduce y_4 and leave y_1.

d_b	c_b	x_b	y_1	y_2	y_3	y_4	Min ratio
2	3	$x_2 = 45/19$	0	1	5/19	-3/19	-15
5	5	$x_1 = 20/19$	1	0	2/19	5/19	4
z^2=209/19	$z^1 = 235/19$	z=0					
		$z_j^1 - c_j$	0	0	5/19	16/19	
		$z_j^2 - d_j$	0	0	0	1	
		Δ_j	_	_	1045/361	-1121/361	

Table 4. Final iteration. Optimal solution.

d_b	c_b	x_b	y_1	y_2	y_3	y_4	Min ratio
2	3	$x_2=3$	3/5	0	1/5	0	
0	0	$y_4=4$	-2/5	1	19/5	0	
$z^2=7$	$z^1=9$	z=9/7					
		$z_j^1-c_j$	-16/5	0	3/5	0	
		$z_j^2-d_j$	-19/5	0	2/5	0	
		Δ_j	59/5	_-	3/5	_	

Now, let us consider the above four tables. We have reached the maximum Z=9/7 and the optimum solution $x_1=0,\ x_2=3$.

5 | Conclusion

In this paper, a new method is proposed to find the fuzzy optimal solution of FLFP problems with inequality constraints. We showed that the method proposed in this paper is very effective. To illustrate the proposed method numerical examples are solved.

References

[1] Tanaka, H., Okuda, T., & Asai, K. (1973). Fuzzy mathematical programming. *Transactions of the society of instrument and control engineers*, *9*(5), 607-613.

[2] Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management science*, *17*(4), B-141. https://doi.org/10.1287/mnsc.17.4.B141

[3] Lai, Y. J., & Hwang, C. L. (1992). A new approach to some possibilistic linear programming problems. *Fuzzy sets and systems*, *49*(2), 121-133.

[4] Shaocheng, T. (1994). Interval number and fuzzy number linear programming. *Fuzzy sets and systems*, *66*(3), 301-306.

[5] Ebrahimnejad, A., & Tavana, M. (2014). A novel method for solving linear programming problems with symmetric trapezoidal fuzzy numbers. *Applied mathematical modelling*, *38*(17-18), 4388-4395.

[6] Lotfi, F. H., Allahviranloo, T., Jondabeh, M. A., & Alizadeh, L. (2009). Solving a full fuzzy linear programming using lexicography method and fuzzy approximate solution. *Applied mathematical modelling*, *33*(7), 3151-3156.

[7] Najafi, H. S., & Edalatpanah, S. A. (2013). A note on "a new method for solving fully fuzzy linear programming problems". *Applied mathematical modelling*, *37*(14-15), 7865-7867.

[8] Dehghan, M., Hashemi, B., & Ghatee, M. (2006). Computational methods for solving fully fuzzy linear systems. *Applied mathematics and computation*, *179*(1), 328-343.