A Comment on "A Novel Parametric Ranking Method for Intuitionistic Fuzzy Numbers"

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Abstract

This note shows that the parametric ranking method proposed by Darehmiraki [M. Darehmiraki, A novel parametric ranking method for intuitionistic fuzzy numbers, Iranian Journal of Fuzzy Systems, 16(1) (2019), 129-143] is not correct. By two appropriate examples, we show that the developed index for ranking intuitionistic fuzzy numbers is not suitable and implies wrong results.

Keywords: Intuitionistic fuzzy number, Ranking, α-cuts, β-cuts.

1 | Introduction

Unlike a fuzzy number which is defined by a membership function, an intuitionistic fuzzy number is described by two functions. One for the degree of belonging of members (membership function) and the other for the degree of not belonging of members (non-membership function) to the set so that the sum of these degrees is not greater than one. By these functions, we can efficiently describe support, opposition, and neutrality (i.e., neither supporting nor opposing) situations. So, intuitionistic fuzzy numbers are more flexible to model uncertainty or fuzziness in real-world applications.

The ranking of intuitionistic fuzzy numbers is one of the main issues in intuitionistic fuzzy set theory [1]. In a recent article, Darehmiraki [2] attempted to introduce a new index for ranking intuitionistic fuzzy numbers by developing Shureshjani and Darehmiraki's [3] work. This comment notifies that the developed index is not meaningful and leads to wrong results. We first show that there are some mistakes in the proposed calculations that we modify them. Then, by two illustrative examples, we prove that the proposed ranking method is not appropriate and leads to wrong ranking results.

2 | Ranking Intuitionistic Fuzzy Numbers

Let $\widehat{y} = (\underline{m}(r), \overline{m}(r), \underline{n}(r), \overline{n}(r))$ be an intuitionistic fuzzy number. The values of the intuitionistic fuzzy number $\widehat{y}$ for membership and non-membership functions are defined as follows (Li [4]):

$$V_{\mu}(\widehat{y}) = \int_{0}^{w_{\widehat{y}}} (\underline{m}(r) + \overline{m}(r)) f(r) dr. \tag{1}$$

And

$$V_{v}(\widehat{y}) = \int_{u_{\widehat{y}}}^{1} (\underline{n}(r) + \overline{n}(r)) g(r) dr. \tag{2}$$

Where $f(r)$ $\left(r \in \left[0, w_{\widehat{y}}\right]\right)$ and $g(r)$ $\left(r \in \left[u_{\widehat{y}}, 1\right]\right)$ satisfy the following conditions:

1) $f(r) \in [0,1]$ $\left(r \in \left[0, w_{\widehat{y}}\right]\right)$ and $g(r) \in [0,1]$ $\left(r \in \left[u_{\widehat{y}}, 1\right]\right)$,

2) $f(0) = 0$ and $g(1) = 0$, and

3) $f(r)$ is monotonic and non-decreasing of $r \in \left[0, w_{\widehat{y}}\right]$ while $g(r)$ is monotonic and non-increasing of $r \in \left[u_{\widehat{y}}, 1\right]$.

Darehmiraki [2] proposed the following ranking intuitionistic fuzzy numbers index:

$$Q_{\alpha, \beta}(\widehat{y}) = Q_{\alpha} - Q_{\beta}, \qquad 0 \leq \alpha \leq w_{\widehat{y}}, \quad u_{\widehat{y}} \leq \beta \leq 1. \tag{3}$$

Where

$$Q_{\alpha} = \int_{\alpha}^{w_{\widehat{y}}} (\underline{m}(r) + \overline{m}(r)) dr, \qquad Q_{\beta} = \int_{u_{\widehat{y}}}^{\beta} (\underline{n}(r) + \overline{n}(r)) dr. \tag{4}$$

For a decision level higher than α and less than β.

As can be seen, Q_{α} and Q_{β} are similar to V_{μ} and V_{v} by considering $f(r) = g(r) = 1$ and α & β instead of 0 & 1, respectively. *Fig.1* shows an arbitrary trapezoidal intuitionistic fuzzy number (TraIFN) $\hat{A}$. The proposed index by Darehmiraki [2] (*Eq. (3)*) graphically is the sum of two dotted areas from α to $w_{\hat{A}}$ minus the sum of two cross-hatched areas from $u_{\hat{A}}$ to β.
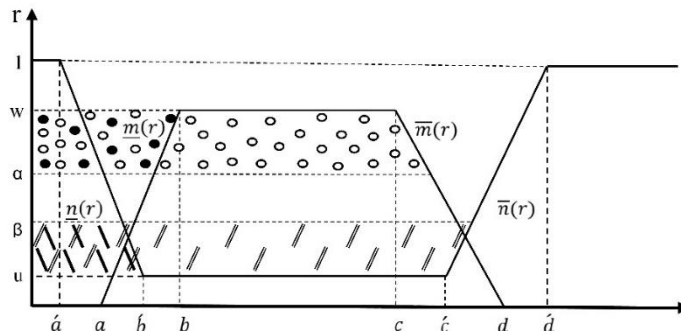

Fig. 1. An arbitrary trapezoidal intuitionistic fuzzy number.

However, in calculating $Q_{\alpha,\beta}^{Tra}$ index for TraIFNs, some calculative mistakes have happened.

Let $\hat{A} = \langle (a_1, a_2, a_3, a_4); w_{\hat{A}}, u_{\hat{A}} \rangle$ be an arbitrary TraIFN. In [2], $Q_{\alpha,\beta}^{Tra}(\hat{A})$ is calculated as below:

$$
\begin{aligned}
Q_{\alpha,\beta}^{Tra}(\widehat{A}) &= \int_\alpha^{w_{\widehat{A}}} (\underline{m}(r) + \overline{m}(r))dr - \int_{u_{\widehat{A}}}^\beta (\underline{n}(r) + \overline{n}(r))dr \\
&= \int_\alpha^{w_{\widehat{A}}} \left(a_1 + \frac{1}{w_{\widehat{A}}}(a_2 - a_1)r + a_4 + \frac{1}{w_{\widehat{A}}}(a_3 - a_4)r \right) dr \\
&\quad + \frac{1}{1-u_{\widehat{A}}} \int_{u_{\widehat{A}}}^\beta \left((r(a_1 - a_2) - u_{\widehat{A}}a_1 + a_2) + (r(a_4 - a_3) + a_3 - u_{\widehat{A}}a_4) \right) dr \\
&= (a_1(w_{\widehat{A}} - \alpha) + \frac{1}{2w_{\widehat{A}}}(a_2 - a_1)(w_{\widehat{A}} - \alpha)^2 + a_4(w_{\widehat{A}} - \alpha) + \frac{1}{2w_{\widehat{A}}}(a_3 \\
&\quad - a_4)(w_{\widehat{A}} - \alpha)^2) - \frac{1}{1-u_{\widehat{A}}}((\frac{(\beta - u_{\widehat{A}})^2}{2}(a_1 - a_2) - u_{\widehat{A}}a_1(\beta - u_{\widehat{A}}) \\
&\quad + a_2(\beta - u_{\widehat{A}})) + (\frac{(\beta - u_{\widehat{A}})^2}{2}(a_4 - a_3) + (\beta - u_{\widehat{A}})(a_3 - a_4u_{\widehat{A}}))).
\end{aligned}
\tag{5}
$$

But, it is clear that in the above-mentioned calculated results, it should appear $(w_{\hat{A}}^2 - \alpha^2)$ instead of $(w_{\hat{A}} - \alpha)^2$.

So, the corrected version will be as follows:

$$
\begin{aligned}
Q_{\alpha,\beta}^{Tra}(\widehat{A}) &= \int_\alpha^{w_{\widehat{A}}} (\underline{m}(r) + \overline{m}(r))dr - \int_{u_{\widehat{A}}}^\beta (\underline{n}(r) + \overline{n}(r))dr \\
&= \int_\alpha^{w_{\widehat{A}}} ((a_1 + (\frac{a_2 - a_1}{w_{\widehat{A}}})r) + (a_4 + (\frac{a_3 - a_4}{w_{\widehat{A}}})r)) \, dr \\
&\quad - \int_{u_{\widehat{A}}}^\beta \left(\left(a_2 + \left(\frac{a_1 - a_2}{1 - u_{\widehat{A}}} \right)(r - u_{\widehat{A}}) \right) + \left(a_3 + \left(\frac{a_4 - a_3}{1 - u_{\widehat{A}}} \right)(r - u_{\widehat{A}}) \right) \right) dr \\
&= a_1(w_{\widehat{A}} - \alpha) + (\frac{a_2 - a_1}{2w_{\widehat{A}}})(w_{\widehat{A}}^2 - \alpha^2) + a_4(w_{\widehat{A}} - \alpha) \\
&\quad + (\frac{a_3 - a_4}{2w_{\widehat{A}}})(w_{\widehat{A}}^2 - \alpha^2) - a_2(\beta - u_{\widehat{A}}) - (\frac{a_1 - a_2}{1 - u_{\widehat{A}}})\frac{(\beta^2 - u_{\widehat{A}}^2)}{2} \\
&\quad + (\frac{a_1 - a_2}{1 - u_{\widehat{A}}})u_{\widehat{A}}(\beta - u_{\widehat{A}}) - a_3(\beta - u_{\widehat{A}}) - (\frac{a_4 - a_3}{1 - u_{\widehat{A}}})\frac{(\beta^2 - u_{\widehat{A}}^2)}{2} \\
&\quad + (\frac{a_4 - a_3}{1 - u_{\widehat{A}}})u_{\widehat{A}}(\beta - u_{\widehat{A}}).
\end{aligned}
\tag{6}
$$

Notice that all of the calculations in the original article should revise according to the above-mentioned modified formula.

Although we have corrected the computational problems, the proposed method has logical flaws yet. The proposed index $(Q_{\alpha,\beta})$ is Q_α minus Q_β. But the use of the minus operator led the calculated index to be irrelevant to the location of the intuitionistic fuzzy number on the coordinate axis. So, the obtained ranking results will be inappropriate. For more explanation, consider the following two illustrative examples. In these examples, the corrected version of $Q_{\alpha,\beta}^{Tra}$ calculation is applied.

3 | Illustrative Examples

Example 1. Let $\hat{A} = \langle(3,4,5,6); 0.8,0.1\rangle$ be a TraIFN (see definition 2.4 of Darehmiraki [2]), then the negative of $\hat{A}$ is $-\hat{A} = \langle(-6,-5,-4,-3); 0.8,0.1\rangle$ (*Fig. 2*).
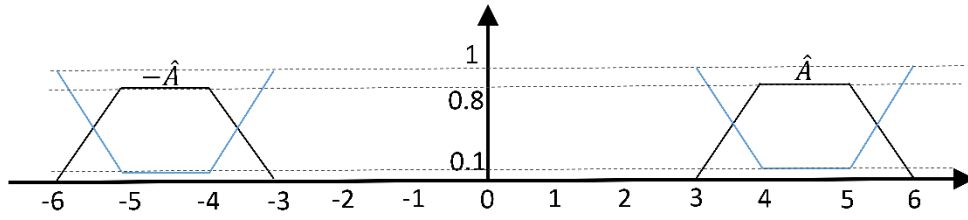

Fig. 2. Two TraIFNs $\hat{A}$ and $-\hat{A}$.

By computing Darehmiraki's index for a decision level higher than 0.7 and lower than 0.3, we have:

$$Q_{0.7,0.3}^{\text{Tra}}\left(\hat{A}\right) = 3(0.8 - 0.7) + \left(\frac{1}{1.6}\right)(0.64 - 0.49) + 6(0.8 - 0.7) + \left(\frac{-1}{1.6}\right)(0.64 - 0.49)$$
$$- 4(0.3 - 0.1) - \left(\frac{-1}{0.9}\right)\frac{(0.09 - 0.01)}{2} + \left(\frac{-1}{0.9}\right)0.1(0.2) - 5(0.2)$$
$$- \left(\frac{1}{0.9}\right)\frac{(0.09 - 0.01)}{2} + \left(\frac{1}{0.9}\right)0.1(0.2) = -0.9.$$

And

$$Q_{0.7,0.3}^{\text{Tra}}\left(-\hat{A}\right) = -6(0.8 - 0.7) + \left(\frac{1}{1.6}\right)(0.64 - 0.49) - 3(0.8 - 0.7) + \left(\frac{-1}{1.6}\right)(0.64 - 0.49)$$
$$+ 5(0.3 - 0.1) - \left(\frac{-1}{0.9}\right)\frac{(0.09 - 0.01)}{2} + \left(\frac{-1}{0.9}\right)0.1(0.2) + 4(0.2)$$
$$- \left(\frac{1}{0.9}\right)\frac{(0.09 - 0.01)}{2} + \left(\frac{1}{0.9}\right)0.1(0.2) = 0.9.$$

So, $Q_{0.7,0.3}^{Tra}\left(\hat{A}\right) < Q_{0.7,0.3}^{Tra}\left(-\hat{A}\right)$ and according to the definition 3.1 in [2], we have $\hat{A} \leq_{0.7,0.3} (-\hat{A})$ that doesn't make sense.

Example 2. Let $\hat{B} = \langle(1,2,3,4); 0.7,0.2\rangle$ be a TraIFN and k $(k > 0)$ be a crisp number, then $\hat{B} + k = \langle(1 + k, 2 + k, 3 + k, 4 + k); 0.7,0.2\rangle$ is also a TraIFN. For a decision level higher than 0.6 and lower than 0.5, we have:

$$Q_{0.6,0.5}^{\text{Tra}}\left(\hat{B}\right) = (0.7 - 0.6) + \left(\frac{1}{1.4}\right)(0.49 - 0.36) + 4(0.7 - 0.6) + \left(\frac{-1}{1.4}\right)(0.49 - 0.36)$$
$$- 2(0.5 - 0.2) - \left(\frac{-1}{0.8}\right)\frac{(0.25 - 0.04)}{2} + \left(\frac{-1}{0.8}\right)0.2(0.5 - 0.2)$$
$$- 3(0.5 - 0.2) - \left(\frac{1}{0.8}\right)\frac{(0.25 - 0.04)}{2} + \left(\frac{1}{0.8}\right)0.2(0.5 - 0.2) = -1.$$

And

$$Q_{0.6,0.5}^{\text{Tra}}\left(\widehat{B} + k\right) = (1 + k)(0.7 - 0.6) + \left(\frac{1}{1.4}\right)(0.49 - 0.36) + (4 + k)(0.7 - 0.6)$$
$$+ \left(\frac{-1}{1.4}\right)(0.49 - 0.36) - (2 + k)(0.5 - 0.2) - \left(\frac{-1}{0.8}\right)\frac{(0.25 - 0.04)}{2}$$
$$+ \left(\frac{-1}{0.8}\right)0.2(0.5 - 0.2) - (3 + k)(0.5 - 0.2) - \left(\frac{1}{0.8}\right)\frac{(0.25 - 0.04)}{2}$$
$$+ \left(\frac{1}{0.8}\right)0.2(0.5 - 0.2) = -1 - 0.4k.$$

So, for all k $(k > 0)$ we have $Q_{0.6,0.5}^{Tra}\left(\widehat{B} + k\right) < Q_{0.6,0.5}^{Tra}\left(\widehat{B}\right)$ and according to the definition 3.1 in [2] we conclude that for all k $(k > 0)$: $\widehat{B} + k \leq_{0.6,0.5} \widehat{B}$ that is incorrect.

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Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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