Big Data and Computing Visions

www.bidacv.com

Big. Data. Comp. Vis. Vol. 3, No. 4 (2023) 196–208.

Paper Type: Original Article

A Theoretical Context for (θ,β)-Convexity and (θ,β)-Concavity with Hypersoft Settings

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Citation:

Ur Rahman, A. (2023). A theoretical context for (θ,β)-convexity and (θ,β)-concavity with hypersoft settings. *Big data and computing visions, 3*(4), 196-208.

Received: 17/03/2023 Reviewed: 19/04/2023 Revised: 09/05/2023 Accept: 10/06/2023

Abstract

Sub-attribute-valued sets are occasionally viewed as more significant in real-life circumstances than a single set of attributes. The current models that deal with ambiguity and uncertainty, or soft sets, are insufficient to address such situations. To adequately fit the current models for multi-attributive sets, the hypersoft set, an extension of the soft set, has been developed. The multi-argument approximate function takes the place of the soft sets' approximate function. Many academics have recently focused on convexity in uncertain environments or soft and fuzzy structures. This paper examines the traditional concepts of (θ, β)-convex and (θ, β)-concave sets in a hypersoft set context, discussing their fundamental inclusive features and set-theoretic operations. Furthermore, traditional notions of first and second senses for convexity are applied to suggested convex structures to provide more broadly applicable outcomes for ambiguous situations.

Keywords: Soft set, Hypersoft set, Convex hypersoft set, Concave hypersoft set, (θ,β)-convex hypersoft set, 52A20, 52A07, 52A99.

1 | Introduction

The idea of a soft set ($\mathbb{SO}$-set) was introduced by Molodtsov [1] in 1999 as a parameterization tool for fuzzy set-like models [2]–[5] that cope with uncertain data. This set uses an approximate function with a multi-valued input that transfers a single set of parameters to the collection of subsets of the set under consideration. The researchers [6]–[13] have examined the fundamentals of s-sets. The authors [14]–[16] produced the hybridized structures of the s-set with a fuzzy set, intuitionistic fuzzy set, and neutrosophic set. Vimala and colleagues [17] investigated the concepts of lattice-based ideals by integrating of $\mathbb{SO}$-set and multifuzzy set. The ranking of airlines during Covid-19 using multi-fuzzy $\mathbb{SO}$-set hybrid-based information was also covered by them [18]. The notions of $\mathbb{SO}$-set are sometimes inadequate in many real-world scenarios where the parameters are to be further classified into their corresponding parametric-valued disjoint sets. This kind of parameter partitioning is beyond the capabilities of the current s-set model. To manage real-life settings, the hypersoft set ($\mathbb{H}y\mathbb{S}$-set) [19] is designed to provide an acceptable s-set for subparametric disjoint sets. The researchers [20], [21]

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https://doi.org/10.22105/bdcv.2023.192676

looked into the fundamental properties and operations of $\mathbb{H}y\mathbb{S}$-set to use it in various knowledge domains.

Many scholars have recently been interested in the *mathbbHymathbbS*-set as an emerging subject of study. To deal with information-based dimensions, Musa et al. [22] expanded $\mathbb{H}y\mathbb{S}$-set to N-Hypersoft Sets. Some forms of $\mathbb{SO}$-set and $\mathbb{H}y\mathbb{S}$-set were introduced by Smarandache [23], [24]. The assessment of enterprise strategic planning utilizing interval-valued neutrosophic $\mathbb{H}y\mathbb{S}$-sets was covered by Zaki and Ismail [25]. Trigonometric similarities between Pythagorean neutrosophic $\mathbb{H}y\mathbb{S}$-sets were developed by Ramya [26]. Arshad et al. [27], [28] examined real estate projects and Covid-19 mask grading using the notions of interval-valued multi-fuzzy $\mathbb{H}y\mathbb{S}$-sets and interval-valued fuzzy $\mathbb{H}y\mathbb{S}$-set, respectively. Rahman et al. [29], [30] used fuzzy parameterization with complex fuzzy $\mathbb{H}y\mathbb{S}$-set and possibly single valued neutrosophic $\mathbb{H}y\mathbb{S}$-set environments to apply mathematical techniques to susceptibility to liver disorder and site selection for solid waste management. Saeed et al. [31] used the notions of fuzzy $\mathbb{H}y\mathbb{S}$-set in graphs. Saeed et al. [32] discussed the various basic notions of interval-valued fuzzy $\mathbb{H}y\mathbb{S}$-sets. Ihsan et al. [33] employed a robust technique to evaluate electronic appliances using an integrated version of the neutrosophic $\mathbb{H}y\mathbb{S}$-set and expert set.

In many mathematical disciplines, convexity and concavity are crucial concepts that offer vital resources for analysis, optimization, and modeling. Understanding convex and concave functions is essential to understanding how derivatives and integrals behave in the calculus domain. Convex functions are essential in optimization issues where the goal is to determine the minimum of a function. They are distinguished by the feature that the line segment joining any two points on the graph lies above it. Conversely, concave functions, which have a reverse characteristic, are used in functions that maximize. Convex geometry goes beyond the scope of calculus to investigate the characteristics of convex sets and how they are used in operations research and linear programming, among other fields. In the field of the field of finance, concave utility coefficients represent decreasing marginal returns, whereas convex inclinations represent logical choices. Convexity and concavity are widely used in the mathematical sciences, emphasizing their importance for comprehending and resolving a wide range of issues. Arshad et al. [34] and Rahman et al. [35] investigated various aspects of classical convexity and concavity in refined neutrosophic and intuitionistic fuzzy set environments, respectively. The researchers [35]–[40] studied the various operational properties and results of classical convexity and concavity in $\mathbb{SO}$-set and its set hybrids environments. Rahman et al. [41] studied the set-based theorems and axioms of classical convexity and concavity in $\mathbb{H}y\mathbb{S}$-set environment.

In this work, we develop (θ,β)-convex and (θ,β)-concave $\mathbb{H}y\mathbb{S}$-sets with their first and second sense generalizations, drawing inspiration from the work presented in [41]. Here, (θ,β) is a real number within a closed unit interval and serves the same function as m in the classical definitions given in [42], [43]. The rest of the paper is structured as provided in *Fig. 1*.

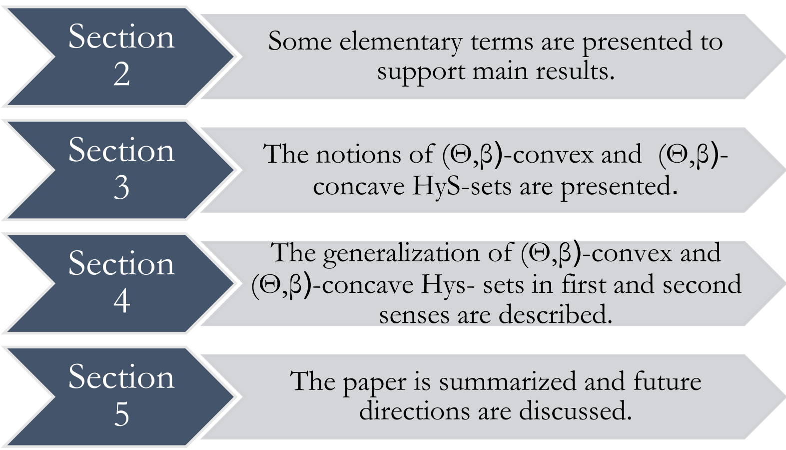

Fig. 1. Structural template of the paper.

2 | Preliminaries

This section reviews a few elementary terminologies regarding s-set and $\mathbb{H}y\$$-set. In the remaining part, $\breve{\mathbb{L}}$, $\widetilde{\mathbb{U}}$ and $\mathbb{I}^{\bullet}$ will denote $\mathbb{R}^n$, the initial universe and [0,1], respectively. [1] If $\mathbb{F}_{\breve{\$}}: \breve{\mathbb{L}} \to P(\widetilde{\mathbb{U}})$ then the pair $\breve{\$} = (\mathbb{F}_{\breve{\$}}, \breve{\mathbb{L}})$ is called s-set where $\breve{\mathbb{L}}$ denotes the collection of attributes. The collection of s-sets is denoted by Ω_{ss}. Some properties of s-set are stated below for proper understanding [6]. If $\breve{\$}$ & $\breve{\mathbb{Q}} \in \Omega_{ss}$ then

I. $\breve{\$} \subseteq \breve{\mathbb{Q}}$ if $\mathbb{F}_{\breve{\$}}(\omega) \subseteq \mathbb{F}_{\breve{\mathbb{Q}}}(\omega)$; *for all* $\omega \in \breve{\mathbb{L}}$.

II. $\breve{\$} \cup \breve{\mathbb{Q}}$ is stated as $\mathbb{F}_{\breve{\$}\cup\breve{\mathbb{Q}}}(\omega) = \mathbb{F}_{\breve{\$}}(\omega) \cup \mathbb{F}_{\breve{\mathbb{Q}}}(\omega)$; *for all* $\omega \in \breve{\mathbb{L}}$.

III. $\breve{\$} \cap \breve{\mathbb{Q}}$ is defined as $\mathbb{F}_{\breve{\$}\cap\breve{\mathbb{Q}}}(\omega) = \mathbb{F}_{\breve{\$}}(\omega) \cap \mathbb{F}_{\breve{\mathbb{Q}}}(\omega)$; *for all* $\omega \in \breve{\mathbb{L}}$.

Please see [6], [7], [9], [11], [13], [15] for operations of s-sets [19]. If $\mathbb{F}_{\breve{\mathbb{H}}}: \mathbb{P} \to P(\widetilde{\mathbb{U}})$ then the set $\breve{\mathbb{H}} = (\mathbb{F}_{\breve{\mathbb{H}}}, \mathbb{P})$ is known as $\mathbb{H}y\$$-set where $\mathbb{P} = \mathbb{P}_1 \times \mathbb{P}_2 \times \mathbb{P}_3 \times \ldots . \times \mathbb{P}_n$ and $\mathbb{P}_i$ are non-overlapping sets having parametric values with respect to different parameters $p_1, p_2, \ldots, p_n$. Please see [20], [21], [27] for more operational properties of $\mathbb{H}y\$$-sets [41]. If $\delta \subseteq \widetilde{\mathbb{U}}$ then the $\delta - inclusion$ of a $\mathbb{H}y\$$-set $\breve{\mathbb{H}}$ is stated as $\breve{\mathbb{H}}^{\delta} = \{\kappa \in \mathbb{P}: \mathbb{F}_{\breve{\mathbb{H}}}(\kappa) \supseteq \delta\}$ [41].

A $\mathbb{H}y\$$-set $\breve{\mathbb{H}}$ on $\mathbb{P}$ is known as a convex $\mathbb{H}y\$$-set if $\mathbb{F}_{\breve{\mathbb{H}}}(a\kappa + (1-a)\tau) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}(\kappa) \cap \mathbb{F}_{\breve{\mathbb{H}}}(\tau)$; for all $\kappa, \tau \in \mathbb{P}$ and $a \in \mathbb{I}^{\bullet}$ [41].

A $\mathbb{H}y\$$-set $\breve{\mathbb{H}}$ on $\mathbb{P}$ is known as concave $\mathbb{H}y\$$-set if $\mathbb{F}_{\breve{\mathbb{H}}}(a\kappa + (1-a)\tau) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}(\kappa) \cup \mathbb{F}_{\breve{\mathbb{H}}}(\tau)$; for all $\kappa, \tau \in \mathbb{P}$ and $a \in \mathbb{I}^{\bullet}$.

3 | The (θ, β)-Convex and (θ, β)-Concave $\mathbb{H}y\$$-Sets

In this part of the paper, the (θ, β)-convex and (θ, β)-concave $\mathbb{H}y\$$-sets are characterized by following the classical definitions from [42], [43] with partial modifications. The $\mathbb{H}y\$$-set $\breve{\mathbb{H}}$ on $\mathbb{P}$ is said to be (θ, β)-convex $\mathbb{H}y\$$-set if

$$\mathbb{F}_{\breve{\mathbb{H}}}(\theta\vartheta_1 + \beta(1-\theta)\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2) \text{ for } \vartheta_1, \vartheta_2 \in \mathbb{P},\ \beta \in \mathbb{I}^{\bullet} \text{ and } \theta \in (0,1]. \quad (1)$$

The collection of all (θ, β)-convex $\mathbb{H}y\$$-sets is represented by $\widehat{\Omega}_{chss}$. In this definition, the set $\mathbb{P}$ is in accordance with *Definition 3*(`)@. If $\breve{\mathbb{H}}$ & $\breve{\mathbb{Q}} \in \widehat{\Omega}_{chss}$, then $\breve{\mathbb{H}} \cap \breve{\mathbb{Q}} \in \widehat{\Omega}_{chss}$.

Proof: Suppose that for $\vartheta_1, \vartheta_2 \in \mathbb{P}$, and $\breve{\mathbb{W}} = \breve{\mathbb{H}} \cap \breve{\mathbb{Q}}$, then

$$\mathbb{F}_{\breve{\mathbb{W}}}(\theta\vartheta_1 + \beta(1-\theta)\vartheta_2) = \mathbb{F}_{\breve{\mathbb{H}}}(\theta\vartheta_1 + \beta(1-\theta)\vartheta_2) \cap \mathbb{F}_{\breve{\mathbb{Q}}}(\theta\vartheta_1 + \beta(1-\theta)\vartheta_2). \quad (2)$$

As $\breve{\mathbb{H}}$ & $\breve{\mathbb{Q}} \in \widehat{\Omega}_{chss}$

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$$\mathbb{F}_{\breve{\mathbb{H}}}(\theta\vartheta_1 + \beta(1-\theta)\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2), \tag{3}$$

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1-\theta)\vartheta_2) = \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta)\vartheta_2) \cap \mathbb{F}_{\breve{\mathbb{Q}}}\ \theta\vartheta_1 + \beta(1-\theta)\vartheta_2), \tag{4}$$

which implies

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1-\theta)\vartheta_2) \supseteq (\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)) \cap (\mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_2)), \tag{5}$$

and thus

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1-\theta)\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{W}}}\ \omega) \cap \mathbb{F}_{\breve{\mathbb{W}}}\ \vartheta_2), \tag{6}$$

$\breve{\mathbb{H}} \in \widehat{\Omega}_{chss}$ on $\mathbb{P}$ iff for every $\theta \in \mathbb{I}^{\bullet}$ and $\delta \in \breve{P}(\widetilde{\mathbb{U}})$, $\breve{\mathbb{H}}^{\delta} \in \widehat{\Omega}_{chss}$ on $\mathbb{P}$.

Proof: Suppose $\breve{\mathbb{H}} \in \widehat{\Omega}_{chss}$. If $\vartheta_1, \vartheta_2 \in \mathbb{P}$ and $\breve{\delta} \in \breve{P}(\widetilde{\mathbb{U}})$, then $\mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_1) \supseteq \breve{\delta}$ and $\mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2) \supseteq \breve{\delta}$

$$\Rightarrow \quad \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2) \supseteq \breve{\delta},$$

$$\Rightarrow \quad \mathbb{F}_{\breve{\mathbb{H}}}\ n\vartheta_1 + \beta\ 1-\theta)\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2) \supseteq \breve{\delta},$$

$$\Rightarrow \quad \mathbb{F}_{\breve{\mathbb{H}}}\ n\vartheta_1 + \beta\ 1-\theta)\vartheta_2) \supseteq \breve{\delta},$$

and thus $\breve{\mathbb{H}}^{\delta} \in \widehat{\Omega}_{chss}$. Conversely, suppose that $\breve{\mathbb{H}}^{\delta} \in \widehat{\Omega}_{chss}$ for every $\theta \in \mathbb{I}^{\bullet}$. For $\vartheta_1, \vartheta_2 \in \mathbb{P}$, $\breve{\mathbb{H}}^{\delta}$ is (θ,β)-convex with $\delta = \mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2)$. Since $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \supseteq \delta$ and $\mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2) \supseteq \breve{\delta}$, we have $\vartheta_1 \in \breve{\mathbb{H}}^{\delta}$ and $\vartheta_2 \in \breve{\mathbb{H}}^{\delta} \Rightarrow \quad n\vartheta_1 + \beta\ 1-\theta)\vartheta_2 \in \breve{\mathbb{H}}^{\delta}$.

Therefore, $\mathbb{F}_{\breve{\mathbb{H}}}\ n\vartheta_1 + m\ 1-\theta)\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2), \Rightarrow \breve{\mathbb{H}} \in \widehat{\Omega}_{chss}$ on $\mathbb{P}$.

A $\mathbb{H}y\mathbb{S}$-set $\breve{\mathbb{H}}$ on $\mathbb{P}$ is known to be (θ,β)-concave $\mathbb{H}y\mathbb{S}$-set if

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta)\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2) \text{ for } \vartheta_1, \vartheta_2 \in \mathbb{P},\ \beta \in \mathbb{I}^{\bullet} \text{ and } \theta \in (0,1]. \tag{7}$$

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The pictorial representations of (θ,β)-convex and (θ,β)-concave $\mathbb{H}y\$$-sets are presented in *Fig. 2*.

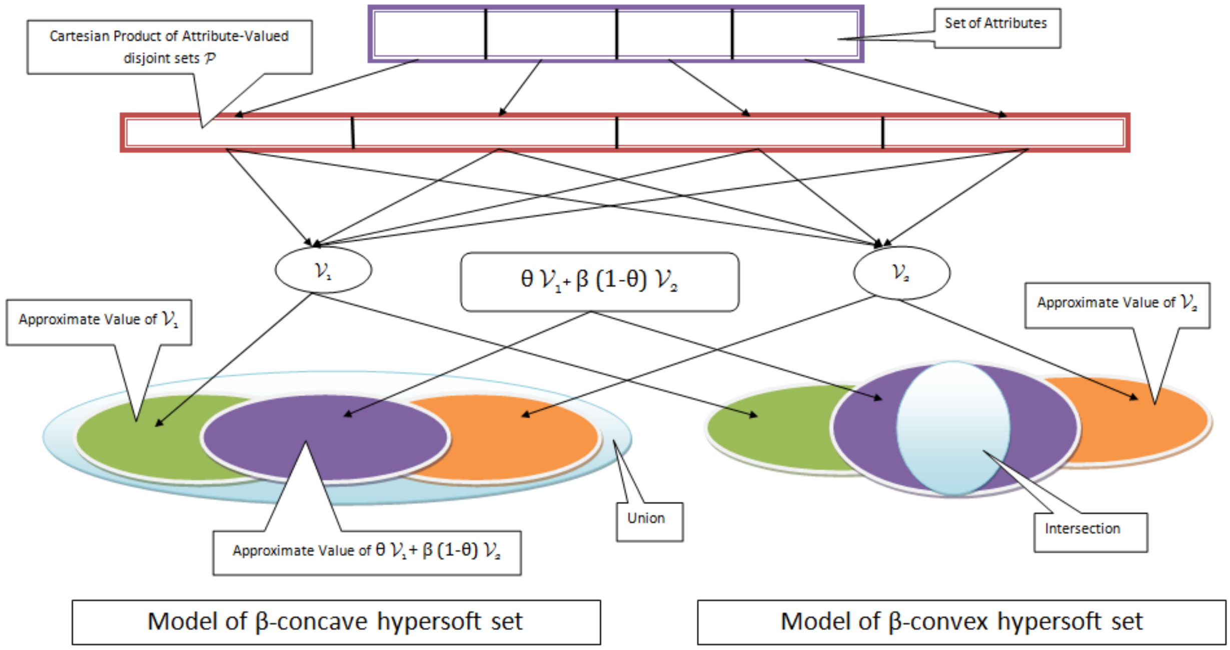

Fig. 2. Pictorial models of (θ,β)-convex and (θ,β)-concave $\mathbb{H}y\$$-sets.

If $\breve{\mathbb{H}}$ & $\breve{\mathbb{Q}} \in \widehat{\Omega}_{cohss}$ then $\breve{\mathbb{H}} \cup \breve{\mathbb{Q}} \in \widehat{\Omega}_{cohss}$.

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2) = \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2) \cup \mathbb{F}_{\breve{\mathbb{Q}}}\ \theta\vartheta_1+\beta\ 1-\theta)\vartheta_2). \quad (8)$$

Proof: Suppose that for $\vartheta_1, \vartheta_2 \in \mathbb{P}$, and $\theta \in \mathbb{I}^{\bullet}$ and $\breve{\mathbb{W}} = \breve{\mathbb{H}} \cup \breve{\mathbb{Q}}$. Then

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2). \quad (9)$$
$$\mathbb{F}_{\breve{\mathbb{Q}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_2), \quad (10)$$

Now, since $\breve{\mathbb{H}}$ and $\breve{\mathbb{Q}}$ are (θ,β)-concave,

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)\ ) \ \cup \left(\mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_2)\right). \quad (11)$$

and hence

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{W}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{W}}}\ \vartheta_2) \text{ If } \breve{\mathbb{H}} \ \& \ \breve{\mathbb{Q}} \in \widehat{\Omega}_{cohss} \text{then} \breve{\mathbb{H}} \cap \breve{\mathbb{Q}} \in \widehat{\Omega}_{cohss}. \quad (12)$$

and thus

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2) = \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2) \cap \mathbb{F}_{\breve{\mathbb{Q}}}\ \theta\vartheta_1+\beta\ 1-\theta)\vartheta_2). \quad (13)$$

Proof: Suppose that for $\vartheta_1, \vartheta_2 \in \mathbb{P}$, and $\theta \in \mathbb{I}^{\bullet}$ and $\breve{\mathbb{W}} = \breve{\mathbb{H}} \cap \breve{\mathbb{Q}}$.

Now, since $\breve{\mathbb{H}}$ and $\breve{\mathbb{Q}}$ are (θ,β)-concave

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2). \quad (14)$$
$$\mathbb{F}_{\breve{\mathbb{Q}}}(\theta\vartheta_1+\beta(1-\theta)\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{Q}}}(\vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{Q}}}(\vartheta_2), \quad (15)$$

and hence,

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2)\subseteq\ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\cup\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2))\cap\Big(\mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_1)\cup\mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_2)\Big),\tag{16}$$

and thus

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2)\subseteq\ \mathbb{F}_{\breve{\mathbb{W}}}\ \vartheta_1)\cup\ \mathbb{F}_{\breve{\mathbb{W}}}\ \vartheta_2)\ \text{If}\ \breve{\mathbb{H}}\in\widehat{\Omega}_{chss}\ \text{then}\ \breve{\mathbb{H}}^{c}\ \in\widehat{\Omega}_{cohss}.\tag{17}$$

Proof: Suppose that for $\vartheta_1,\vartheta_2\in\mathbb{P},\ \theta\in\mathbb{I}^{\bullet}$, and $\breve{\mathbb{H}}$ be (θ,β)-convex $\mathbb{H}y\$$-set.

Since $\breve{\mathbb{H}}$ is (θ,β)-convex,

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2)\supseteq\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\cap\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2),\tag{18}$$

or

$$\widetilde{\mathbb{U}}\backslash\ \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2)\subseteq\widetilde{\mathbb{U}}\backslash\Big\{\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\cap\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)\Big\}.\tag{19}$$

If $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\supset\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$ then $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\cap\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)\ =\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$ then, $\widetilde{\mathbb{U}}\backslash\ \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2)\subseteq\widetilde{\mathbb{U}}\backslash\ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$.

If $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\subset\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$ then $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\cap\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)=\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)$,

Then we may write

$$\widetilde{\mathbb{U}}\backslash\ \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2)\subseteq\widetilde{\mathbb{U}}\backslash\ \ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1),\tag{20}$$

so we have

$$\widetilde{\mathbb{U}}\backslash\ \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2)\subseteq\Big\{\widetilde{\mathbb{U}}\backslash\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\cup\widetilde{\mathbb{U}}\backslash\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)\Big\},\tag{21}$$

which shows that $\breve{\mathbb{H}}^{c}$ is (θ,β)-concave $\mathbb{H}y\$$-set.

If $\breve{\mathbb{H}}\in\widehat{\Omega}_{cohss}$ then $\breve{\mathbb{H}}^{c}\ \in\widehat{\Omega}_{chss}$.

Proof: Suppose that for $\vartheta_1,\vartheta_2\in\mathbb{P},\ \theta\in\mathbb{I}^{\bullet}$, and $\breve{\mathbb{H}}$ be (θ,β)-concave $\mathbb{H}y\$$-set.

Since $\breve{\mathbb{H}}$ is (θ,β)-concave,

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2)\subseteq\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\cup\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2),\tag{22}$$

or

$$\widetilde{\mathbb{U}}\backslash\ \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2)\supseteq\widetilde{\mathbb{U}}\backslash\Big\{\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\cup\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)\Big\}.\tag{23}$$

If $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\supset\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$ then $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\cup\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)=\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)$ then, $\widetilde{\mathbb{U}}\backslash\ \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2)\supseteq\widetilde{\mathbb{U}}\backslash\ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)$.

If $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\subset\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$ then $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\cup\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)=\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$, then we may write

$$\widetilde{\mathbb{U}}\backslash\ \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2)\supseteq\widetilde{\mathbb{U}}\backslash\ \ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2),\tag{24}$$

so we have

$$\widetilde{\mathbb{U}}\backslash\ \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1+\beta(1-\theta)\vartheta_2)\supseteq\Big\{\widetilde{\mathbb{U}}\backslash\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)\cap\widetilde{\mathbb{U}}\backslash\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)\Big\}.\tag{25}$$

So, $\breve{\mathbb{H}}^{c}$ is (θ,β)-convex $\mathbb{H}y\$$-set.

$\breve{\mathbb{H}}\in\widehat{\Omega}_{cohss}$ on $\mathbb{P}$ iff for every $\theta\in\ \mathbb{I}^{\bullet}$ and $\delta\in\ \breve{P}\big(\widetilde{\mathbb{U}}\big)$, $\breve{\mathbb{H}}^{\delta}\in\widehat{\Omega}_{cohss}$ on $\mathbb{P}$.

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Proof: Suppose that $\breve{\mathbb{H}}$ is (θ,β)-concave $\mathbb{H}y\mathbb{S}$-set. If $\vartheta_1,\vartheta_2 \in \mathbb{P}$ and $\breve{\delta} \in \breve{P}(\widetilde{\mathbb{U}})$, then $\mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_1) \supseteq \breve{\delta}$ and $\mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2) \supseteq \breve{\delta}$ then $\mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2) \supseteq \breve{\delta}$. It can explored from (θ,β)-concavity of $\breve{\mathbb{H}}$ that

$$\breve{\delta} \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \mathrm{n}\vartheta_1 + \beta\ 1-\theta)\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2),$$

$$\Rightarrow \breve{\delta} \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta\ 1-\theta)\vartheta_2),$$

$$\Rightarrow \breve{\mathbb{H}}^{\breve{\delta}} \in \widehat{\Omega}_{\mathrm{cohss}}.$$

Conversely, let $\breve{\mathbb{H}}^{\delta} \in \widehat{\Omega}_{cohss}$ for each $\theta \in \mathbb{I}^{\bullet}$. For $\vartheta_1,\vartheta_2 \in \mathbb{P}$, $\breve{\mathbb{H}}^{\delta}$ is concave with $\delta = \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$. Since $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \subseteq \delta$ and $\mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2) \subseteq \breve{\delta}$, we have $\vartheta_1 \in \breve{\mathbb{H}}^{\delta}$ and $\vartheta_2 \in \breve{\mathbb{H}}^{\delta}$, hence $\theta\vartheta_1 + \beta\ 1-\theta)\vartheta_2 \in \breve{\mathbb{H}}^{\delta}$. $\Rightarrow$ $\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta\ 1-\theta)\vartheta_2) \subseteq \breve{\mathbb{H}}^{\delta}$.

Therefore, $\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta\ 1-\theta)\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$, which proves the (θ,β)-concavity of $\breve{\mathbb{H}}$ on $\breve{\mathrm{X}}$.

4 | The (θ,β)-Convex and (θ,β)-Concave $\mathbb{H}$y$-Sets in First and Second Senses

In this segment, two classical approaches, i.e., 1st and 2nd senses, which assign special conditions on (θ,β)-convex and (θ,β)-concave $\mathbb{H}y\mathbb{S}$-sets, are investigated. The $\mathbb{H}y\mathbb{S}$-set $\breve{\mathbb{H}}$ on $\mathbb{P}$ is called a (θ,β)-convex $\mathbb{H}y\mathbb{S}$-set in the first sense if

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta^{\eta})\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2), \tag{26}$$

for $\vartheta_1,\vartheta_2 \in \mathbb{P}$, $\beta \in \mathbb{I}^{\bullet}$ and $\eta,\theta \in (0,1]$. The collection of these sets is denoted by $(\widehat{\Omega}_{chss})^1$. The $\mathbb{H}y\mathbb{S}$-set $\breve{\mathbb{H}}$ on $\mathbb{P}$ is called a (θ,β)-convex $\mathbb{H}y\mathbb{S}$-set in a second sense if

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta)^{\eta}\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2), \tag{27}$$

for $\vartheta_1,\vartheta_2 \in \mathbb{P}$, $\beta \in \mathbb{I}^{\bullet}$ and $\eta,\theta \in (0,1]$. The collection of these sets is denoted by $(\widehat{\Omega}_{chss})^2$ If $\breve{\mathbb{H}}$ & $\breve{\mathbb{Q}} \in (\widehat{\Omega}_{chss})^1$ then $\breve{\mathbb{H}} \cap \breve{\mathbb{Q}} \in (\widehat{\Omega}_{chss})^1$.

Proof: Suppose that for $\vartheta_1,\vartheta_2 \in \mathbb{P}$, and $\breve{\mathbb{W}} = \breve{\mathbb{H}} \cap \breve{\mathbb{Q}}$, then,

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1-\theta^{\eta})\vartheta_2) = \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta^{\eta})\vartheta_2) \cap \mathbb{F}_{\breve{\mathbb{Q}}}\ \theta\vartheta_1 + \beta(1-\theta^{\eta})\vartheta_2). \tag{28}$$

Since $\breve{\mathbb{H}}$ & $\breve{\mathbb{Q}} \in (\widehat{\Omega}_{chss})^1$,

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta^\eta)\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2), \quad (29)$$

$$\mathbb{F}_{\breve{\mathbb{Q}}}\ \theta\vartheta_1 + \beta(1-\theta^\eta)\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_2). \quad (30)$$

then

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1-\theta^\eta)\vartheta_2) \supseteq \left(\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)\right) \cap \left(\mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_2)\right), \quad (31)$$

thus

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1-\theta^\eta)\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{W}}}\ \omega) \cap \mathbb{F}_{\breve{\mathbb{W}}}\ \vartheta_2). \quad (32)$$

If $\breve{\mathbb{H}}$ & $\breve{\mathbb{Q}} \in (\widehat{\Omega}_{chss})^2$ then $\breve{\mathbb{H}} \cap \breve{\mathbb{Q}} \in (\widehat{\Omega}_{chss})^2$.

Proof: Suppose that for $\vartheta_1, \vartheta_2 \in \mathbb{P}$, and $\breve{\mathbb{W}} = \breve{\mathbb{H}} \cap \breve{\mathbb{Q}}$. Then,

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1-\theta)^\eta\vartheta_2) = \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta)^\eta\vartheta_2) \cap \mathbb{F}_{\breve{\mathbb{Q}}}\ \theta\vartheta_1 + \beta(1-\theta)^\eta\vartheta_2). \quad (33)$$

Since $\breve{\mathbb{H}}$ & $\breve{\mathbb{Q}} \in (\widehat{\Omega}_{chss})^2$,

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta)^\eta\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2). \quad (34)$$

$$\mathbb{F}_{\breve{\mathbb{Q}}}\ \theta\vartheta_1 + \beta(1-\theta)^\eta\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_2), \quad (35)$$

which implies

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1-\theta)^\eta\vartheta_2) \supseteq \left(\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)\right) \cap \left(\mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_2)\right), \quad (36)$$

and thus

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1-\theta)^\eta\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{W}}}\ \omega) \cap \mathbb{F}_{\breve{\mathbb{W}}}\ \vartheta_2), \quad (37)$$

$\breve{\mathbb{H}} \in (\widehat{\Omega}_{chss})^1$ on $\mathbb{P}$ iff for every $\theta \in \mathbb{I}^\bullet$ and $\delta \in \breve{P}(\widetilde{\mathbb{U}})$, $\breve{\mathbb{H}}^\delta \in (\widehat{\Omega}_{chss})^1$ on $\mathbb{P}$.

Proof: Consider $\breve{\mathbb{H}} \in (\widehat{\Omega}_{chss})^1$. If $\vartheta_1, \vartheta_2 \in \mathbb{P}$, and $\delta \in \breve{P}(\widetilde{\mathbb{U}})$, then $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \supseteq \delta$ and $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2) \supseteq \delta$. It follows from (θ,β)-convexity of $\breve{\mathbb{H}}$ that

$$\mathbb{F}_{\breve{\mathbb{H}}}\ n\vartheta_1 + \beta\ 1-\theta^\eta)\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2) \Rightarrow \mathrm{M}^{\breve{\delta}} \in (\widehat{\Omega}_{\mathrm{chss}})^1. \quad (38)$$

Conversely, let $\breve{\mathbb{H}}^\delta \in (\widehat{\Omega}_{chss})^1$ for every $\theta \in \mathbb{I}^\bullet$. For $\vartheta_1, \vartheta_2 \in \mathbb{P}$, $\breve{\mathbb{H}}^{\breve{\delta}}$ is (θ,β)-convex for $\delta = \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$.

Since $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \supseteq \delta$ and $\mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2) \supseteq \breve{\delta}$, we have $\vartheta_1 \in \breve{\mathbb{H}}^\delta$ and $\vartheta_2 \in \breve{\mathbb{H}}^\delta$,

$$\Rightarrow \theta\vartheta_1 + \beta\ 1-\theta^\eta)\vartheta_2 \in \breve{\mathbb{H}}^{\breve{\delta}},$$

$$\Rightarrow \mathbb{F}_{\breve{\mathbb{H}}}\ n\vartheta_1 + \beta\ 1-\theta^\eta)\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2),$$

$$\Rightarrow \breve{\mathbb{H}} \in (\widehat{\Omega}_{\mathrm{chss}})^1 \text{ on } \mathbb{P},$$

$\breve{\mathbb{H}} \in (\widehat{\Omega}_{chss})^2$ on $\mathbb{P} \Leftrightarrow$ for every $\theta \in \mathbb{I}^\bullet$ and $\delta \in \breve{P}(\widetilde{\mathbb{U}})$, $\breve{\mathbb{H}}^\delta \in (\widehat{\Omega}_{chss})^2$ on $\mathbb{P}$.

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Proof: Let $\breve{\mathbb{H}} \in (\widehat{\Omega}_{chss})^2$. If $\vartheta_1, \vartheta_2 \in \mathbb{P}$ and $\delta \in \breve{P}(\widetilde{\mathbb{U}})$, then $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \supseteq \delta$ and $\mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2) \mathbin{\breve{\supseteq}} \breve{\delta}$. The (θ,β)-convexity of $\breve{\mathbb{H}}$ implies that

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta\ 1 - \theta)^{\eta}\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2) \Rightarrow \breve{\mathbb{H}}^{\breve{\delta}} \in (\widehat{\Omega}_{\mathrm{chss}})^2. \tag{39}$$

Conversely, let $\breve{\mathbb{H}}^{\delta} \in (\widehat{\Omega}_{chss})^2$ for every $\delta \in \mathbb{I}^{\bullet}$. For $\vartheta_1, \vartheta_2 \in \mathbb{P}$, $\breve{\mathbb{H}}^{\delta}$ is (θ,β)-convex for $\delta = \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$. Since $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \supseteq \delta$ and $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2) \supseteq \delta$, we have $\vartheta_1 \in \breve{\mathbb{H}}^{\delta}$ and $\vartheta_2 \in M^{\delta}$,

$$\Rightarrow \theta\vartheta_1\ + \beta\ 1 - \theta)^{\eta}\vartheta_2 \in\ \breve{\mathbb{H}}^{\breve{\delta}},$$

$$\Rightarrow \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta\ 1 - \theta)^{\eta}\vartheta_2) \supseteq\ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2),$$

$$\Rightarrow \breve{\mathbb{H}} \in (\widehat{\Omega}_{\mathrm{chss}})^2 \text{ on } \mathbb{P}.$$

A $\mathbb{H}y\$$-set $\breve{\mathbb{H}}$ on $\mathbb{P}$ is known as (θ,β)-concave $\mathbb{H}y\$$-set in 1st sense if

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1 - \theta^{\eta})\vartheta_2) \subseteq\ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2), \tag{40}$$

for every $\vartheta_1, \vartheta_2 \in \mathbb{P}$, $\beta \in \mathbb{I}^{\bullet}$ and $\eta, \theta \in (0,1]$. The collection of such sets is represented by $(\widehat{\Omega}_{cohss})^1$. A $\mathbb{H}y\$$-set $\breve{\mathbb{H}}$ on $\mathbb{P}$ is said to be (θ,β)-concave $\mathbb{H}y\$$-set in 2nd sense if

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1 - \theta)^{\eta}\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2), \tag{41}$$

for every $\vartheta_1, \vartheta_2 \in \mathbb{P}$, $\beta \in \mathbb{I}^{\bullet}$ and $\eta, \theta \in (0,1]$. The collection of such sets is represented by $(\widehat{\Omega}_{cohss})^2$. If $\breve{\mathbb{H}}\ \&\ \breve{\mathbb{Q}} \in (\widehat{\Omega}_{cohss})^1$ then $\breve{\mathbb{H}} \cap \breve{\mathbb{Q}} \in (\widehat{\Omega}_{cohss})^1$.

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1 - \theta^{\eta})\vartheta_2) = \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1 - \theta^{\eta})\vartheta_2) \cap\ \mathbb{F}_{\breve{\mathbb{Q}}}\ \theta\vartheta_1 + \beta(1 - \theta^{\eta})\vartheta_2). \tag{42}$$

Proof: Let for $\vartheta_1, \vartheta_2 \in \mathbb{P}$, and $\theta \in \mathbb{I}^{\bullet}$ and $\breve{\mathbb{W}} = \breve{\mathbb{H}} \cap \breve{\mathbb{Q}}$. Then,

Since $\breve{\mathbb{H}}\ \&\ \breve{\mathbb{Q}} \in (\widehat{\Omega}_{cohss})^1$, then

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1 - \theta^{\eta})\vartheta_2) \subseteq\ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup\ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2), \tag{43}$$

$$\mathbb{F}_{\breve{\mathbb{Q}}}\ \theta\vartheta_1 + \beta(1 - \theta^{\eta})\vartheta_2) \subseteq\ \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_1) \cup\ \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_2). \tag{44}$$

And

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1 - \theta^{\eta})\vartheta_2) \subseteq\ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)\ )\ \cap \left(\mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_2)\right), \tag{45}$$

and thus

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1 - \theta^{\eta})\vartheta_2) \subseteq\ \mathbb{F}_{\breve{\mathbb{W}}}\ \vartheta_1) \cup\ \mathbb{F}_{\breve{\mathbb{W}}}\ \vartheta_2). \tag{46}$$

If $\breve{\mathbb{H}}\ \&\ \breve{\mathbb{Q}} \in (\widehat{\Omega}_{\mathrm{cohss}})^2$ then $\breve{\mathbb{H}} \cap \breve{\mathbb{Q}} \in (\widehat{\Omega}_{cohss})^2$.

Proof: Consider for $\vartheta_1, \vartheta_2 \in \mathbb{P}$, and $\theta \in \mathbb{I}^{\bullet}$ and $\breve{\mathbb{W}} = \breve{\mathbb{H}} \cap \breve{\mathbb{Q}}$. Then,

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1 - \theta)^{\eta}\vartheta_2) =\ \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1 - \theta)^{\eta}\vartheta_2) \cap \mathbb{F}_{\breve{\mathbb{Q}}}\ \theta\vartheta_1 + \beta(1 - \theta)^{\eta}\vartheta_2). \tag{47}$$

Since $\breve{\mathbb{H}}$ & $\breve{\mathbb{Q}} \in (\widehat{\Omega}_{cohss})^2$,

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta)^{\eta}\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2), \tag{48}$$

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1-\theta)^{\eta}\vartheta_2) = \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta)^{\eta}\vartheta_2) \cap \mathbb{F}_{\breve{\mathbb{Q}}}\ \theta\vartheta_1 + \beta(1-\theta)^{\eta}\vartheta_2), \tag{49}$$

and

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1-\theta)^{\eta}\vartheta_2) \subseteq \left(\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)\right) \cap \left(\mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{Q}}}\ \vartheta_2)\right), \tag{50}$$

and

$$\mathbb{F}_{\breve{\mathbb{W}}}\ \theta\vartheta_1 + \beta(1-\theta)^{\eta}\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{W}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{W}}}\ \vartheta_2). \tag{51}$$

If $\breve{\mathbb{H}} \in (\widehat{\Omega}_{chss})^2$ then $\breve{\mathbb{H}}^c \in (\widehat{\Omega}_{cohss})^2$.

Proof: Let for $\vartheta_1, \vartheta_2 \in \mathbb{P}$, $\theta \in \mathbb{I}^\bullet$, and $\breve{\mathbb{H}} \in (\widehat{\Omega}_{chss})^2$.

then,

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta)^{\eta}\vartheta_2) \supseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2), \tag{52}$$

or

$$\widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta)^{\eta}\vartheta_2) \subseteq \widetilde{\mathbb{U}} \setminus \left\{\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cap \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)\right\}. \tag{53}$$

If $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \supset \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$ then

$$\widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta)^{\eta}\vartheta_2) \subseteq \widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2). \tag{54}$$

If $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \subset \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$ then

$$\widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta)^{\eta}\vartheta_2) \subseteq \widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1). \tag{55}$$

From the above equations, we have

$$\widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta)^{\eta}\vartheta_2) \subseteq (\widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)) \cup (\widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)) \Rightarrow \breve{\mathbb{H}}^c \in (\widehat{\Omega}_{chss})^2. \tag{56}$$

If $\breve{\mathbb{H}} \in (\widehat{\Omega}_{cohss})^1$ then $\breve{\mathbb{H}}^c \in (\widehat{\Omega}_{chss})^1$.

Proof: Suppose that there exist $\vartheta_1, \vartheta_2 \in \mathbb{P}$, $\theta \in \mathbb{I}^\bullet$ and $\breve{\mathbb{H}} \in (\widehat{\Omega}_{cohss})^1$.

then,

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta^{\eta})\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2), \tag{57}$$

or

$$\widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta^{\eta})\vartheta_2) \supseteq \widetilde{\mathbb{U}} \setminus \left\{\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)\right\}. \tag{58}$$

If $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \supset \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$ then

$$\widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta^{\eta})\vartheta_2) \supseteq \widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1). \tag{59}$$

If $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \subset \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$ then

$$\widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta^{\eta})\vartheta_2) \supseteq \widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2). \tag{60}$$

From *Eq. (24)* and *(25)*, we have

$$\widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta(1-\theta^{\eta})\vartheta_2) \supseteq (\widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1)) \cap (\widetilde{\mathbb{U}} \setminus \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)) \Rightarrow S^c \in (\widehat{\Omega}_{cohss})^1, \tag{61}$$

$\breve{\mathbb{H}} \in (\widehat{\Omega}_{cohss})^1$ on $\mathbb{P} \Leftrightarrow$ for every $\theta \in \mathbb{I}^\bullet$ and $\delta \in \breve{P}(\widetilde{\mathbb{U}}), \breve{\mathbb{H}}^\delta \in (\widehat{\Omega}_{cohss})^1$ on $\mathbb{P}$.

Proof: Let $\breve{\mathbb{H}} \in (\widehat{\Omega}_{cohss})^1$. If $\vartheta_1, \vartheta_2 \in \mathbb{P}$ and $\delta \in \breve{P}(\widetilde{\mathbb{U}})$, then $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \supseteq \delta$ and $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2) \supseteq \delta$. The (θ,β)-concavity of $\breve{\mathbb{H}}$ in 1st sense implies that

$$\mathbb{F}_{\breve{\mathbb{H}}}\ n\vartheta_1 + \beta\ 1 - \theta^{\eta})\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2) \Rightarrow \breve{\mathbb{H}}^{\breve{\delta}} \in (\widehat{\Omega}_{\mathrm{cohss}})^1. \tag{62}$$

Conversely, let $\breve{\mathbb{H}}^{\delta} \in (\widehat{\Omega}_{cohss})^1$ for every $\delta \in \mathbb{I}^{\bullet}$. For $\vartheta_1, \vartheta_2 \in \mathbb{P}$, $\breve{\mathbb{H}}^{\breve{\delta}}$ is (θ,β)-concave for $\delta = \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$. Since $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \subseteq \delta$ and $\mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2) \breve{\subseteq} \breve{\delta}$, we have $\vartheta_1 \in \breve{\mathbb{H}}^{\delta}$ and $\vartheta_2 \in \breve{\mathbb{H}}^{\delta}$,

$$\Rightarrow n\vartheta_1\ + \beta\ 1 - \theta^{\eta})\vartheta_2 \in\ \breve{\mathbb{H}}^{\delta},$$

$$\Rightarrow \mathbb{F}_{\breve{\mathbb{H}}}\ n\vartheta_1 + \beta\ 1 - \theta^{\eta})\vartheta_2) \subseteq\ \delta =\ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2),$$

$$\Rightarrow \breve{\mathbb{H}} \in (\widehat{\Omega}_{cohss})^1 \text{ on } \mathbb{P}.$$

$\breve{\mathbb{H}} \in (\widehat{\Omega}_{cohss})^2$ on $\mathbb{P} \Leftrightarrow$ for every $\theta \in \mathbb{I}^{\bullet}$ and $\delta \in\ \breve{P}(\widetilde{\mathbb{U}}), \breve{\mathbb{H}}^{\delta} \in (\widehat{\Omega}_{cohss})^2$ on $\mathbb{P}$.

Proof: Assume that $\breve{\mathbb{H}} \in (\widehat{\Omega}_{cohss})^2$. If $\vartheta_1, \vartheta_2 \in \mathbb{P}$ and $\delta \in\ \breve{P}(\breve{U})$, then $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \supseteq \delta$ and $\mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2) \breve{\supseteq} \breve{\delta}$. By (θ,β)-concavity of $\breve{\mathbb{H}}$, we get

$$\mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta\ 1 - \theta)^{\eta}\vartheta_2) \subseteq \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2) \Rightarrow \breve{\mathbb{H}}^{\breve{\delta}} \in (\widehat{\Omega}_{\mathrm{cohss}})^2. \tag{63}$$

Conversely, consider $\breve{\mathbb{H}}^{\delta} \in (\widehat{\Omega}_{cohss})^2$ for every $\theta \in \mathbb{I}^{\bullet}$. For $\vartheta_1, \vartheta_2 \in \mathbb{P}$, $\breve{\mathbb{H}}^{\delta} \in (\widehat{\Omega}_{cohss})^2$ for $\delta = \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2)$. Since $\mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \subseteq \delta$ and $\mathbb{F}_{\breve{\mathbb{H}}}(\vartheta_2) \breve{\subseteq} \breve{\delta}$, we have $\vartheta_1 \in M^{\delta}$ and $\vartheta_2 \in M^{\delta}$,

$$\Rightarrow n\vartheta_1\ + \beta\ 1 - \theta)^{\eta}\vartheta_2 \in\ \breve{\mathbb{H}}^{\breve{\delta}},$$

$$\Rightarrow \mathbb{F}_{\breve{\mathbb{H}}}\ \theta\vartheta_1 + \beta\ 1 - \theta)^{\eta}\vartheta_2) \subseteq\ \breve{\delta} =\ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_1) \cup\ \mathbb{F}_{\breve{\mathbb{H}}}\ \vartheta_2), \tag{64}$$

$$\Rightarrow \breve{\mathbb{H}} \in (\widehat{\Omega}_{\mathrm{cohss}})^2 \text{ on } \mathbb{P}.$$

5 | Conclusion

This study develops $\mathbb{H}y\$$-sets that are (θ,β)-convex and (θ,β)-concave, as well as their generalized set-axiomatic operations. Furthermore, to obtain more generalized versions of (θ,β)-convex and (θ,β)-concave $\mathbb{H}y\$$-sets, several conventional techniques, namely first and second senses, are applied. In the future, this study can be extended to uncertain settings to introduce various types of convexity, such as triangular and graded convexity.

Data Availability Statement

"Not applicable".

Ethical Considerations

The author avoided data fabrication and falsification.

Conflict of Interest

The author declare no conflict of interest.

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